

TABLE OF INTEGRALS

BASIC FORMS

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| <p>1. $\int u \, dv = uv - \int v \, du$</p> <p>2. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$</p> <p>3. $\int \frac{du}{u} = \ln u + C$</p> <p>4. $\int e^u \, du = e^u + C$</p> <p>5. $\int a^u \, du = \frac{a^u}{\ln a} + C$</p> <p>6. $\int \sin u \, du = -\cos u + C$</p> <p>7. $\int \cos u \, du = \sin u + C$</p> <p>8. $\int \sec^2 u \, du = \tan u + C$</p> <p>9. $\int \csc^2 u \, du = -\cot u + C$</p> <p>10. $\int \sec u \tan u \, du = \sec u + C$</p> | <p>11. $\int \csc u \cot u \, du = -\csc u + C$</p> <p>12. $\int \tan u \, du = \ln \sec u + C$</p> <p>13. $\int \cot u \, du = \ln \sin u + C$</p> <p>14. $\int \sec u \, du = \ln \sec u + \tan u + C$</p> <p>15. $\int \csc u \, du = \ln \csc u - \cot u + C$</p> <p>16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$</p> <p>17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$</p> <p>18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$</p> <p>19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left \frac{u+a}{u-a} \right + C$</p> <p>20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left \frac{u-a}{u+a} \right + C$</p> |
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FORMS INVOLVING $\sqrt{a^2 + u^2}, \quad a > 0$

21. $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
22. $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$
23. $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$
24. $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$
25. $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$
26. $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
27. $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
28. $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29. $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$

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FORMS INVOLVING $\sqrt{a^2 - u^2}$, $a > 0$

$$30. \int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$31. \int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$32. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$33. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$$

$$34. \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$35. \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$36. \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$$

$$37. \int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$38. \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

FORMS INVOLVING $\sqrt{u^2 - a^2}$, $a > 0$

$$39. \int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$40. \int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$41. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$$

$$42. \int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$$

$$43. \int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$$

$$44. \int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$45. \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$46. \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$

TABLE OF INTEGRALS

FORMS INVOLVING $\sqrt{a^2 - u^2}$, $a > 0$

$$30. \int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$31. \int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$32. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$33. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$$

$$34. \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$35. \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$36. \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$$

$$37. \int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$38. \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

FORMS INVOLVING $\sqrt{u^2 - a^2}$, $a > 0$

$$39. \int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$40. \int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$41. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$$

$$42. \int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$$

$$43. \int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$$

$$44. \int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$45. \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$46. \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$

TABLE OF INTEGRALS

FORMS INVOLVING $a + bu$

$$47. \int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$$

$$48. \int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$$

$$49. \int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$$

$$50. \int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$51. \int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$$

$$52. \int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$53. \int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left(a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$$

$$54. \int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$$

$$55. \int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$$

$$56. \int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$$

$$57. \int \frac{du}{u \sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \quad \text{if } a > 0$$

$$= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \quad \text{if } a < 0$$

$$58. \int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u \sqrt{a + bu}}$$

$$59. \int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u \sqrt{a + bu}}$$

$$60. \int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$$

$$61. \int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$$

$$62. \int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n - 1)u^{n-1}} - \frac{b(2n - 3)}{2a(n - 1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$$

TABLE OF INTEGRALS

TRIGONOMETRIC FORMS

63. $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4} \sin 2u + C$
64. $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4} \sin 2u + C$
65. $\int \tan^2 u \, du = \tan u - u + C$
66. $\int \cot^2 u \, du = -\cot u - u + C$
67. $\int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$
68. $\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$
69. $\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln |\cos u| + C$
70. $\int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln |\sin u| + C$
71. $\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$
72. $\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$
73. $\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$
74. $\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$
75. $\int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du$
76. $\int \cot^n u \, du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$
77. $\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$
78. $\int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$
79. $\int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$
80. $\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$
81. $\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$
82. $\int u \sin u \, du = \sin u - u \cos u + C$
83. $\int u \cos u \, du = \cos u + u \sin u + C$
84. $\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$
85. $\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$
86. $\int \sin^n u \cos^m u \, du = \frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u \, du$
 $= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u \, du$

INVERSE TRIGONOMETRIC FORMS

87. $\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$
88. $\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$
89. $\int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$
90. $\int u \sin^{-1} u \, du = \frac{2u^2-1}{4} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$
91. $\int u \cos^{-1} u \, du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$
92. $\int u \tan^{-1} u \, du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$
93. $\int u^n \sin^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$
94. $\int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$
95. $\int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{1+u^2} \right], \quad n \neq -1$

TABLE OF INTEGRALS

EXPONENTIAL AND LOGARITHMIC FORMS

$$96. \int u e^{au} du = \frac{1}{a^2} (au - 1)e^{au} + C$$

$$97. \int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$98. \int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$$

$$99. \int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$$

$$100. \int \ln u du = u \ln u - u + C$$

$$101. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$102. \int \frac{1}{u \ln u} du = \ln |\ln u| + C$$

HYPERBOLIC FORMS

$$103. \int \sinh u du = \cosh u + C$$

$$104. \int \cosh u du = \sinh u + C$$

$$105. \int \tanh u du = \ln \cosh u + C$$

$$106. \int \coth u du = \ln |\sinh u| + C$$

$$107. \int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$$

$$108. \int \operatorname{csch} u du = \ln \left| \tanh \frac{1}{2} u \right| + C$$

$$109. \int \operatorname{sech}^2 u du = \tanh u + C$$

$$110. \int \operatorname{csch}^2 u du = -\coth u + C$$

$$111. \int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$$

$$112. \int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$$

FORMS INVOLVING $\sqrt{2au - u^2}$, $a > 0$

$$113. \int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$114. \int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$115. \int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$116. \int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$117. \int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$118. \int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$119. \int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u+3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$120. \int \frac{du}{u\sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$